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On dimensional reductions of the M-9-brane

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Abstract

We discuss the relations of the M-9-brane with other branes via dimensional reductions, mainly focusing on their Wess-Zumino (WZ) actions. It is shown that on three kinds of dimensional reductions, the WZ action of the M-9-brane respectively gives those of the D-8-brane, the "KK-8A brane" (which we regard as a kind of D-8-brane) and the "NS-9A brane", the last two actions of which were obtained via dualities. Based on these results, we conclude that the relation of p-branes for $p \geq 8$, proposed previously, is consistent from the viewpoint of worldvolume actions.

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1 Introduction

M-theory is a candidate for a unified theory of particle interactions and is conjectured to be the 11-dimensional (11D) theory [1][2] which gives 5 perturbative 10-dimensional (10D) string theories in different kinds of limits. In discussing properties of these theories, (p+1)-dimensional objects, called p-branes, play many crucial roles (e.g. [3]), so, it is important to clarify what kinds of branes exist in each of the theories. Brane scan via superalgebra is one of the methods to discuss them, in which one can read BPS branes possible to exist in the theories from the structure of central charges of superalgebras[4][5]. For $p \leq 7$, all of the BPS branes predicted from superalgebras to exist in M-, IIA and IIB string theories have corresponding solutions in each of the supergravities i.e. the low energy limits of the theories. However, the p-branes we want to discuss in this paper are those with $p \geq 8$, for which one kind of 9-brane is suggested to exist in M-theory[4], one kinds of 8-brane and 9-branes are predicted in IIA, and two kinds of 9-branes are in IIB[4]. The first one is called "M-9-brane", and the others are called (or identified with) D-8-brane, NS-9A-brane, D-9-brane and NS-9B-brane, respectively, based on the consideration of the kinds of charges they are supposed to have[4]. Taking into account the dimensions and the duality relations of the theories, the relation of the p-branes for $p \geq 8$, suggested from superalgebras, are represented as Figure 1[4] (see also [9]). ("KK8A" will be explained below.) These branes are very important in that M- and string theories with 16 supercharges are expected to be constructed by using these branes[3][6][7][4][8][10] (see Figure 2).† We will discuss these branes.

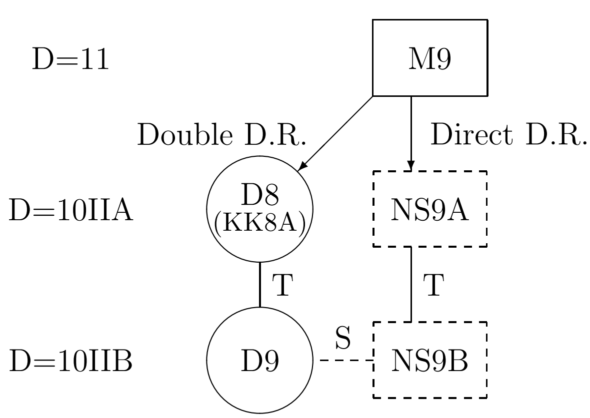

Figure 1: Relations of p-branes with $p \geq 8$

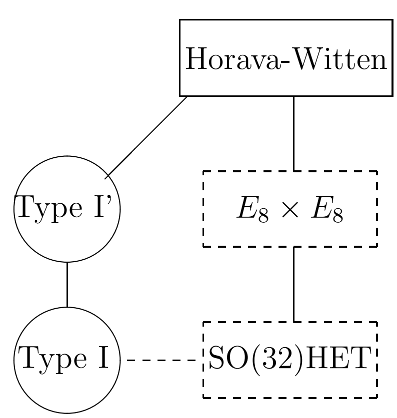

Figure 2: M and strings with 16 SUSY

However, before presenting our work, we have to mention the following problem with

† "Horava-Witten" in Figure 2 denotes the Horava-Witten construction of M-theory[11].

the M-9-brane, or 11D origins of the D-8-brane solution and massive IIA SUGRA, and related issues. The BPS D-8-brane arising in the IIA string, actually, has a corresponding solution not in the usual IIA but in the massive IIA SUGRA with non-vanishing cosmological term[3][6][12]. This is because a BPS D-8-brane in 10 dimensions is a domain wall with some electric charge of a RR 9-form gauge field, giving rise to a constant field strength, which we denote as a mass parameter m in this paper. This field strength contributes to the action as the cosmological constant $-m^2/2$.‡ In other words, such domain wall solutions cannot be constructed without cosmological term. In 11D, however, no deformation to include a cosmological term is allowed if Riemannian geometry and covariant action are assumed[14]. Thus, there is no naive M-9-brane solution in 11D, and the origin of the D-8-brane and massive IIA SUGRA are still unclear.

There are several approaches to solve this problem[15][16][17][18], and one of them is "massive 11D theory"[17]. This is a trial theory, constructed on the basis of the idea that *the problem may imply the need to modify the framework of 11D SUGRA*. Suppose a Killing isometry is assumed in the 11D background. Then, the no-go theorem is avoided and the massive 11D theory, which is written in terms of an 11-dimensional theory at least formally, can be defined; it gives the 10D massive IIA SUGRA on dimensional reduction along the isometry direction (which is parametrized by the coordinate z), and gives usual 11D SUGRA in the massless limit $m \to 0$ if the dependence of the fields on z is restored. Moreover, the M-9-brane solution, i.e. the solution which gives a D-8-brane solution on the dimensional reduction along z, is obtained in this theory[12] (see also [20]). We note that only the bosonic sectors have been discussed in this massive 11D theory, though its bulk theory is called "super"gravity. We also note that the isometry direction is interpreted as a compactified direction like S^1, and the M-9-brane is considered to be wrapped around it[12] (see also [9]).§ We follow the above idea and study the relations of branes within this framework.

Then, there arises a further issue on dimensional reductions of the M-9-brane, due to the existence of the isometry; if one dimensionally reduce the M-9-brane along the worldvolume direction different from the isometry, it is considered to give an 8-brane with one isometry direction as a worldvolume one. This brane is called "KK-8A brane", whose properties have been discussed in ref.[23][24][25] (see also [9]): The KK-8A brane is described as a solution of another massive IIA SUGRA which is obtained by reducing

‡ In addition, the mass term of the NSNS 2-form B arises in the action of IIA SUGRA in this case[13][12], so, the theory is called "massive" theory and the corresponding background is called massive background.

§ More detailed target-space picture of the M-9-branes have been discussed in ref.[19][10]. The tension of the M-9-brane is discussed in ref.[9][19]. For K-theoretic discussion on M-9-brane, see ref.[21]. For brane decent relations including the M-9-brane, see ref.[22].

the massive 11D SUGRA in a direction different from the isometry one[23], and the KK-8A brane is conjectured to be the T-dual of the "NS-7B brane" which is the S-dual of the D-7-brane.[23][24][25]. The problem is, *to what extent we should take it as an object independent of the D-8-brane*; if one considers the introduction of the Killing vector to be some intrinsic modification of the theory and regards the isometry direction as a certain special one, this is surely the third possibility of the dimensional reduction of the M-9-brane. On the other hand, if the isometry originates from some M-theory background (for example, the one considered by Hull in ref.[18]), it is not the third possibility and the KK-8A brane solution should be regarded as a kind of description of the D-8-brane in some special background. In ref.[25] it is argued that from the viewpoint of target-space solutions, the two 8-branes represent the same physical object since the solutions relate to each other via a certain coordinate transformation¶ (although from the viewpoint of spacetime superalgebra, the KK-8A brane should be different from the D-8-brane). Based on this argument, we consider that the isometry emerges from some background, as discussed in ref.[10], and discuss the KK-8A brane as a kind of D-8-brane. (In addition, we will give a certain comment on the relation of the KK-8A brane and superalgebras in the final section.)

To be concrete, we will discuss the relations from the viewpoint of worldvolume effective actions (WVEAs). In fact, all the WVEAs of the bosonic sectors of branes have been constructed[26][27][28][17][29][19][30][24][25][20]. In particular, the worldvolume actions of the NS-9A brane and the KK-8A brane (as a D-8-brane) have been obtained *via chains of dualities.*‖ However, the relations of the action of the M-9-brane with those of the D-8-brane and the NS-9A brane *via dimensional reductions* have not been discussed for the full action; the discussion were done only for their kinetic actions[24], merely because the WZ action of the M-9-brane has been obtained only recently[20].** In other words, *even within this framework, the consistency of the relation of the branes has not been established yet.*

The purpose of this paper is to examine the consistency of the relations of the p-branes with $p \geq 8$ in Fig.1 from the viewpoint of WVEAs, by discussing the dimensional reductions of the M-9-brane WZ action and comparing them with the WZ actions of the D-8-brane and the NS-9A brane.

The organization of this paper is as follows: In section 2, we first review the massive

¶See (the pages 33 and 34 in section 6 of the hep-th version of) ref.[25].

‖ To be concrete, the action of the KK-8A brane has been obtained from that of the D-7-brane by S- and T-dualizing it[25].

**The relation of the WZ action of the M-9-brane with that of the D-8-brane was discussed partly in ref.[20], only for the case of the dimensional reduction along z.

11D SUGRA and construction of the M-9-brane WZ action. A certain logical mistake in ref.[20] in constructing it is corrected. (To be concrete, the action should be constructed only on the basis of its gauge invariance, but should not be so constructed to give the D-8-brane WZ action on dimensional reduction.) In section 3 we discuss three kinds of dimensional reductions of the action, and compare them with the actions of the D-8-brane, the KK-8A brane and the NS-9A brane, respectively. In section 4 we give summary and discussion, especially on the massive part of the KK-8A brane WZ action, and the relation of the KK-8A branes with spacetime superalgebras.

The notation of this paper is as follows: We use "mostly-minus" metrics for both target-spaces and worldvolumes. Fields, indices and coordinates with hats are 11-dimensional, while those with no hats are 10-dimensional. We denote spacetime coordinates by $\hat{x}^{\hat{\mu}}, \hat{x}^{\hat{\nu}}, \cdots$ or $x^{\mu}, x^{\nu}, \cdots$, and local Lorentz indices by $\hat{a}, \hat{b}, \cdots$ or $a, b, \cdots$. Finally, we set $2\pi\alpha' = 1$.

2 The massive 11D SUGRA and the M-9-brane WZ action

2. 1 Review of the massive 11D SUGRA

In this section we first review the massive 11D supergravity[17][20]. The bosonic field content of the supergravity is the same as that of the usual (massless) 11D supergravity: the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ and the 3-form gauge potential $\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$. In this theory these fields are required to have a Killing isometry, i.e., $\mathcal{L}_{\hat{k}}\hat{g}_{\hat{\mu}\hat{\nu}} = \mathcal{L}_{\hat{k}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 0$ where $\mathcal{L}_{\hat{k}}$ indicates a Lie derivative with respect to a Killing vector field $\hat{k}^{\hat{\mu}}$. (The coordinates are so chosen that the isometry direction is parametrized by the coordinate $\hat{x}^z = z$, i.e. $\hat{k}^{\hat{\mu}} = \hat{\delta}^{\hat{\mu}z}$.) The infinitesimal gauge transformations of the fields are defined as[17]

$$\delta\hat{g}_{\hat{\mu}\hat{\nu}} = -m[\hat{\lambda}_{\hat{\mu}}(i_{\hat{k}}\hat{g})_{\hat{\nu}} + \hat{\lambda}_{\hat{\nu}}(i_{\hat{k}}\hat{g})_{\hat{\mu}}], \tag{2.1}$$

$$\delta\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3\hat{\partial}_{[\hat{\mu}}\hat{\chi}^{(2)}_{\hat{\nu}\hat{\rho}]} - 3m\hat{\lambda}_{[\hat{\mu}}(i_{\hat{k}}\hat{C})_{\hat{\nu}\hat{\rho}]} \tag{2.2}$$

where $(i_{\hat{k}}\hat{T}^{(r)}_{\hat{\mu}_1\cdots\hat{\mu}_{r-1}}) \equiv \hat{k}^{\hat{\mu}}\hat{T}^{(r)}_{\hat{\mu}_1\cdots\hat{\mu}_{r-1}\hat{\mu}}$ for a field $\hat{T}^{(r)}$. $\hat{\chi}^{(2)}_{\hat{\mu}\hat{\nu}}$ is the infinitesimal 2-form gauge parameter, and $\hat{\lambda}_{\hat{\mu}}$ is defined as $\hat{\lambda}_{\hat{\mu}} \equiv (i_{\hat{k}}\hat{\chi}^{(2)})_{\hat{\mu}}$.* We note that the transformation corresponding to $\hat{\lambda}$ is called "massive gauge transformation". Then, a connection for the massive gauge transformations must be introduced. The new total connection takes the form $\hat{\Omega}_{\hat{a}}{}^{\hat{b}\hat{c}} = \hat{\omega}_{\hat{a}}{}^{\hat{b}\hat{c}} + \hat{K}_{\hat{a}}{}^{\hat{b}\hat{c}}$ where $\hat{\omega}_{\hat{a}}{}^{\hat{b}\hat{c}}$ is a usual spin connection and $\hat{K}$ is given by[17]

$$\hat{K}_{\hat{a}}{}^{\hat{b}\hat{c}} = \frac{m}{2}[\hat{k}_{\hat{a}}(i_{\hat{k}}\hat{C})^{\hat{b}\hat{c}} + \hat{k}^{\hat{b}}(i_{\hat{k}}\hat{C})_{\hat{a}}{}^{\hat{c}} - \hat{k}^{\hat{c}}(i_{\hat{k}}\hat{C})_{\hat{a}}{}^{\hat{b}}]. \tag{2.3}$$

*In this paper we change the notation of ref.[17] such that $m \to 2m$ and $\hat{\lambda} \to -\frac{1}{2}\hat{\lambda}$.

The 4-form field strength $\hat{G}^{(4)}$ of $\hat{C}$ is defined as[17]

$$\hat{G}^{(4)}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = 4\hat{D}_{[\hat{\mu}}\hat{C}_{\hat{\nu}\hat{\rho}\hat{\sigma}]} \equiv 4\hat{\partial}_{[\hat{\mu}}\hat{C}_{\hat{\nu}\hat{\rho}\hat{\sigma}]} + 3m(i_{\hat{k}}\hat{C})_{[\hat{\mu}\hat{\nu}}(i_{\hat{k}}\hat{C})_{\hat{\rho}\hat{\sigma}]} \tag{2.4}$$

where $\hat{D}_{\hat{\mu}}$ denotes the covariant derivative. Then, $\hat{G}^{(4)}$ transforms covariantly as

$$\delta\hat{G}^{(4)}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = 4m\hat{\lambda}_{[\hat{\mu}}(i_{\hat{k}}\hat{G}^{(4)})_{\hat{\nu}\hat{\rho}\hat{\sigma}]}, \tag{2.5}$$

which implies that $\delta(\hat{G}^{(4)})^2 = 0$.

The action of the massive 11D supergravity is[17]

$$\begin{aligned} \hat{S}_0 &= \frac{1}{\hat{\kappa}}\int d^{11}\hat{x}[\sqrt{|\hat{g}|}\{\hat{R} - \frac{1}{2\cdot 4!}(\hat{G}^{(4)})^2 + \frac{1}{2}m^2|\hat{k}|^4\} \\ &+ \frac{\hat{\epsilon}^{\hat{\mu}_1\cdots\hat{\mu}_{11}}}{(144)^2}\{2^4\hat{\partial}\hat{C}\hat{\partial}\hat{C}\hat{C} + 18m\hat{\partial}\hat{C}\hat{C}(i_{\hat{k}}\hat{C})^2 + \frac{3^3}{5}m^2\hat{C}(i_{\hat{k}}\hat{C})^4\}_{\hat{\mu}_1\cdots\hat{\mu}_{11}}] \end{aligned} \tag{2.6}$$

where $\hat{\kappa} = 16\pi G_N^{(11)}$ and $|\hat{k}| = \sqrt{-\hat{k}^{\hat{\mu}}\hat{k}^{\hat{\nu}}\hat{g}_{\hat{\mu}\hat{\nu}}}$.† This action is invariant up to total derivative under the gauge transformation (2.1) and (2.2).

The 10-form gauge potential $\hat{A}^{(10)}$ to which the M-9-brane minimally couples is introduced[20] by promoting the mass parameter m to a scalar field $\hat{M}(\hat{x})$, and adding to the action $\hat{S}_0$ the extra term

$$\Delta\hat{S} = \frac{1}{\hat{\kappa}}\int d^{11}\hat{x}\frac{1}{11!}\hat{\epsilon}^{\hat{\mu}_1\cdots\hat{\mu}_{11}}\hat{M}(\hat{x})11\hat{\partial}_{[\hat{\mu}_1}\hat{A}^{(10)}_{\hat{\mu}_2\cdots\hat{\mu}_{11}]}. \tag{2.7}$$

At this moment, the gauge transformation of the original action $\hat{S}_0$ does not vanish but is proportional to $\hat{\partial}\hat{M}$. So, the total action $\hat{S}^{\text{total}} \equiv \hat{S}_0 + \Delta\hat{S}$ becomes invariant under (2.1) and (2.2) if the massive gauge transformation of $\hat{A}^{(10)}$ is defined as[20]

$$\begin{aligned} \delta(i_{\hat{k}}\hat{A}^{(10)})_{\hat{\mu}_1\cdots\hat{\mu}_9} &= -\sqrt{|\hat{g}|}\hat{\epsilon}_{\hat{\mu}_1\cdots\hat{\mu}_9\hat{\mu}z}[-\hat{g}^{\hat{\mu}\hat{\mu}'}\hat{g}^{\hat{\nu}\hat{\nu}'}(2\hat{\partial}_{[\hat{\mu}'}\hat{k}_{\hat{\nu}']} - \hat{M}|\hat{k}|^2(i_{\hat{k}}\hat{C})_{\hat{\mu}'\hat{\nu}'})\hat{\lambda}_{\hat{\nu}} \\ &+ \frac{1}{2}\hat{G}^{(4)\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}(i_{\hat{k}}\hat{C})_{\hat{\nu}\hat{\rho}}\hat{\lambda}_{\hat{\sigma}}] - \frac{9!}{48}[\hat{\partial}\hat{C}(i_{\hat{k}}\hat{C})^2\hat{\lambda} + \frac{\hat{M}}{4}(i_{\hat{k}}\hat{C})^4\hat{\lambda}]_{\hat{\mu}_1\cdots\hat{\mu}_9}. \end{aligned} \tag{2.8}$$

We note that the 10-form $\hat{A}^{(10)}$ with *no* z index does not arise in the theory because $\hat{A}^{(10)}$ enters the theory through the additional action (2.7) and that $\hat{A}^{(10)}$ satisfies $\mathcal{L}_{\hat{k}}\hat{A}^{(10)} = 0$.

† By using the (generalized) Palatini's identity given in ref.[17], the action (2.6) is rewritten as

$$\begin{aligned} \hat{S}_0 &= \frac{1}{\hat{\kappa}}\int d^{11}\hat{x}[\sqrt{|\hat{g}|}\{-\hat{\Omega}_{\hat{b}}{}^{\hat{b}\hat{a}}\hat{\Omega}_{\hat{c}}{}^{\hat{c}}{}_{\hat{a}} - \hat{\Omega}_{\hat{a}}{}^{\hat{b}\hat{c}}\hat{\Omega}_{\hat{b}\hat{c}}{}^{\hat{a}} - \frac{1}{2\cdot 4!}(\hat{G}^{(4)})^2 + \frac{1}{2}m^2|\hat{k}^2|^2\}\} \\ &+ \frac{1}{144}\hat{\epsilon}^{\hat{\mu}_1\cdots\hat{\mu}_{10}z}\{\hat{\partial}\hat{C}\hat{\partial}\hat{C}(i_{\hat{k}}\hat{C}) + \frac{m}{2}\hat{\partial}\hat{C}(i_{\hat{k}}\hat{C})^3 + \frac{9m^2}{80}(i_{\hat{k}}\hat{C})^5\}_{\hat{\mu}_1\cdots\hat{\mu}_{10}z} + (\text{surface terms})]. \end{aligned}$$

In fact, the 10D massive IIA action in ref.[12] is obtained from this action only if the surface terms are omitted. Omitting them, we use this action as a "starting" action, in order to make the correspondence of the 11D theory with the 10D one.

We also note that the 10-form is not a dynamical field (even if $\hat{M}$ is integrated out and the term like its kinetic term arises in the action) in the same way as the case of the RR 9-form potential in 10 dimensions (see ref.[3]).

In order to construct the gauge invariant M-9-brane WZ action (and to derive an appropriate expression of field strength of $\hat{A}^{(10)}$), dual fields of the 3-form $\hat{C}$ and a "1-form potential" $\hat{k}_{\hat{\mu}}$ need to be introduced[20].‡ They are the 6-form and the 8-form potentials $\hat{C}^{(6)}$ and $\hat{N}^{(8)}$, to which the M-5-brane and M-KK-monopole minimally couple[17][29], respectively. Their gauge transformations for a constant mass background $\hat{M} = m$ are defined as[17]

$$\begin{aligned}
\delta\hat{C}^{(6)}_{\hat{\mu}_1\cdots\hat{\mu}_6} &= 6\hat{\partial}_{[\hat{\mu}_1}\hat{\chi}^{(5)}_{\hat{\mu}_2\cdots\hat{\mu}_6]} + 30\hat{\partial}_{[\hat{\mu}_1}\hat{\chi}^{(2)}_{\hat{\mu}_2\hat{\mu}_3}\hat{C}_{\hat{\mu}_4\hat{\mu}_5\hat{\mu}_6]} + 6m\hat{\lambda}_{[\hat{\mu}_1}(i_{\hat{k}}\hat{C}^{(6)})_{\hat{\mu}_2\cdots\hat{\mu}_6]} && (2.9)\\
\delta\hat{N}^{(8)}_{\hat{\mu}_1\cdots\hat{\mu}_8} &= \{8\hat{\partial}\hat{\Omega}^{(7)} + 168\hat{\partial}\hat{\chi}^{(5)}(i_{\hat{k}}\hat{C}) + \frac{8!}{3\cdot 4!}\hat{\partial}\hat{\chi}^{(2)}\hat{C}(i_{\hat{k}}\hat{C}) + 8m\hat{\lambda}(i_{\hat{k}}\hat{N}^{(8)})\}_{[\hat{\mu}_1\cdots\hat{\mu}_8]} && (2.10)
\end{aligned}$$

where $\hat{\chi}^{(5)}$ and $\hat{\Omega}^{(7)}$ are 5-form and 7-form parameters of the gauge transformation associated with $\hat{C}^{(6)}$ and $\hat{N}^{(8)}$, respectively. The duality relations which they satisfy are[17][20]

$$\hat{G}^{(4)\hat{\mu}_1\cdots\hat{\mu}_4} = \frac{\hat{\epsilon}^{\hat{\mu}_1\cdots\hat{\mu}_{11}}}{7!\sqrt{|\hat{g}|}}\hat{G}^{(7)}_{\hat{\mu}_5\cdots\hat{\mu}_{11}}, \quad \hat{G}^{(2)\hat{\mu}_1\hat{\mu}_2} = \frac{\hat{\epsilon}^{\hat{\mu}_1\cdots\hat{\mu}_{10}z}}{9!\sqrt{|\hat{g}|}}(i_{\hat{k}}\hat{G}^{(9)})_{\hat{\mu}_3\cdots\hat{\mu}_{10}} \qquad (2.11)$$

where

$$\begin{aligned}
\hat{G}^{(4)}_{\hat{\mu}_1\cdots\hat{\mu}_4} &\equiv \{4\hat{\partial}\hat{C} + 3m(i_{\hat{k}}\hat{C})(i_{\hat{k}}\hat{C})\}_{\hat{\mu}_1\cdots\hat{\mu}_4} && (2.12)\\
\hat{G}^{(7)}_{\hat{\mu}_1\cdots\hat{\mu}_7} &\equiv 7\{\hat{\partial}\hat{C}^{(6)} - 3m(i_{\hat{k}}\hat{C})(i_{\hat{k}}\hat{C}^{(6)}) + 10\hat{C}\hat{\partial}\hat{C} \\
&\quad +5m\hat{C}(i_{\hat{k}}\hat{C})^2 + \frac{m}{7}(i_{\hat{k}}\hat{N}^{(8)})\}_{\hat{\mu}_1\cdots\hat{\mu}_7} && (2.13)\\
\hat{G}^{(2)}_{\hat{\mu}\hat{\nu}} &\equiv 2\hat{\partial}_{[\hat{\mu}}\hat{k}_{\hat{\nu}]} - m|\hat{k}|^2(i_{\hat{k}}\hat{C})_{\hat{\mu}\hat{\nu}} && (2.14)\\
(i_{\hat{k}}\hat{G}^{(9)})_{\hat{\mu}_1\cdots\hat{\mu}_8} &\equiv 8\{\hat{\partial}(i_{\hat{k}}\hat{N}^{(8)}) + 21(i_{\hat{k}}\hat{C}^{(6)})\hat{\partial}(i_{\hat{k}}\hat{C}) \\
&+ 35\hat{C}\hat{\partial}(i_{\hat{k}}\hat{C})(i_{\hat{k}}\hat{C}) + 35\hat{\partial}\hat{C}(i_{\hat{k}}\hat{C})^2 + \frac{105}{8}m(i_{\hat{k}}\hat{C})^4\}_{[\hat{\mu}_1\cdots\hat{\mu}_8]} && (2.15)
\end{aligned}$$

are the field strengths of $\hat{C}, \hat{C}^{(6)}, \hat{k}_\mu$ and $i_{\hat{k}}\hat{N}^{(8)}$, respectively.§

By using (2.11), the gauge invariant WZ action of the M-9-brane is constructed[20]. The field strength of $\hat{A}^{(10)}$ can also be defined, but in order to discuss dimensional reductions, it is convenient to introduce a 10-form potential $\hat{C}^{(10)}$ which gives usual RR 9-form potential $C^{(9)}$ in the massive IIA theory on dimensional reduction along z. Based

‡The problem was that the first part of (2.8) cannot be expressed as a sum of products of forms, but we can rewrite the part into that if dual fields are used appropriately through duality relations.

§ In the case of $\hat{N}^{(8)}$, its full field strength is difficult to construct, but that of $i_{\hat{k}}\hat{N}^{(8)}$ can be obtained, and it is sufficient for the present purpose[20].

on the transformation property under the massive transformation, $\hat{C}^{(10)}$ is identified as the following redefined field:

$$
\begin{aligned}
(i_{\hat{k}}\hat{C}^{(10)})_{\hat{\mu}_1\cdots\hat{\mu}_9} \;\equiv\; & (i_{\hat{k}}\hat{A}^{(10)})_{\hat{\mu}_1\cdots\hat{\mu}_9} + 9!\,[\frac{1}{2\cdot 7!}(i_{\hat{k}}\hat{N}^{(8)})(i_{\hat{k}}\hat{C}) \\
& -\frac{1}{2^3\cdot 5!}(i_{\hat{k}}\hat{C}^{(6)})(i_{\hat{k}}\hat{C})^2 + \frac{1}{2^4\cdot(3!)^2}\hat{C}(i_{\hat{k}}\hat{C})^3]_{[\hat{\mu}_1\cdots\hat{\mu}_9]}.
\end{aligned}
\tag{2.16}
$$

The 11-form field strength $\hat{G}^{(11)}$ of $\hat{C}^{(10)}$ is given as the equation of motion for $\hat{M}$, as

$$
\frac{\delta\hat{S}^{total}}{\delta\hat{M}} = 0 \;\Leftrightarrow\; \hat{M}|\hat{k}|^4 = -*\hat{G}^{(11)}
\tag{2.17}
$$

where $*$ means Hodge or Poincare dual, and

$$
\begin{aligned}
\hat{G}^{(11)} = (i_{\hat{k}}\hat{G}^{(11)})_{\hat{\mu}_1\cdots\hat{\mu}_{10}} \;=\; & 10\{\hat{\partial}(i_{\hat{k}}\hat{C}^{(10)}) - 36\hat{\partial}(i_{\hat{k}}\hat{C})(i_{\hat{k}}\hat{N}^{(8)}) \\
& -36\cdot 35\hat{C}\hat{\partial}(i_{\hat{k}}\hat{C})(i_{\hat{k}}\hat{C})^2 + \frac{189}{2}m(i_{\hat{k}}\hat{C})^5\}_{\hat{\mu}_1\cdots\hat{\mu}_{10}}.
\end{aligned}
\tag{2.18}
$$

Then, the gauge transformations of $i_{\hat{k}}\hat{C}^{(10)}$ can be defined so as to keep $\hat{G}^{(11)}$ invariant[20], as

$$
\begin{aligned}
\delta(i_{\hat{k}}\hat{C}^{(10)})_{\hat{\mu}_1\cdots\hat{\mu}_9} \;=\; & 9!\{\frac{1}{8!}\hat{\partial}(i_{\hat{k}}\hat{\Omega}^{(9)}) + \frac{1}{2\cdot 6!}\hat{\partial}(i_{\hat{k}}\hat{\Omega}^{(7)})(i_{\hat{k}}\hat{C}) + \frac{1}{2^3\cdot 4!}\hat{\partial}(i_{\hat{k}}\hat{\chi}^{(5)})(i_{\hat{k}}\hat{C})^2 \\
+\; & \frac{1}{2^4\cdot 3!}\hat{\partial}\hat{\chi}^{(2)}(i_{\hat{k}}\hat{C})^3 - \frac{1}{2^4\cdot 4!}m\hat{\lambda}(i_{\hat{k}}\hat{C})^4\}_{[\hat{\mu}_1\cdots\hat{\mu}_9]},
\end{aligned}
\tag{2.19}
$$

where $\hat{\Omega}^{(9)}$ is a 9-form parameter of the gauge transformation associated with $\hat{C}^{(10)}$.

2. 2 Review of the M-9-brane WZ action

In this approach, the M-9-brane wrapped around the compact isometry direction is described[19] (see also [9]), and its worldvolume effective action is constructed as that of a gauged σ-model[19][29][20], where the the translation along $\hat{k}^{\hat{\mu}}$ is gauged[17] (see also ref.[31]). Denoting its worldvolume coordinates by ξ^i $(i = 0, 1, .., 8)$ and their embeddings by $\hat{X}^{\hat{\mu}}(\xi)$ $(\hat{\mu} = 0, 1, \cdots, 9, z)$, the worldvolume gauge transformation is written by

$$
\delta_\eta \hat{X}^{\hat{\mu}} = \eta(\xi)\hat{k}^{\hat{\mu}},
\tag{2.20}
$$

where $\eta(\xi)$ is a scalar gauge parameter. To make the brane action invariant under the transformation, the derivative of $\hat{X}^{\hat{\mu}}$ with respect to ξ^i is replaced by the covariant derivative $D_i\hat{X}^{\hat{\mu}} = \partial_i\hat{X}^{\hat{\mu}} - \hat{A}_i\hat{k}^{\hat{\mu}}$ with the gauge field $\hat{A}_i = -|\hat{k}|^{-2}\partial_i\hat{X}^{\hat{\nu}}\hat{k}_{\hat{\nu}}$[31]. ($D_i\hat{X}^{\hat{\mu}}$ is invariant under (2.20).) Then, *only on the basis of the invariance under the massive gauge transformation* (and (2.20)), the M-9-brane WZ action (for a constant mass background) is

constructed as[20]

$$\begin{aligned}
S^{WZ}_{M9} \quad &= \quad \int d^9\xi\hat{\epsilon}^{i_1\cdots i_9}[\frac{1}{9!}(i_{\hat{k}}\widetilde{\hat{C}^{(10)}})_{i_1\cdots i_9} + \frac{1}{2\cdot 7!}(i_{\hat{k}}\widetilde{\hat{N}^{(8)}})_{i_1\cdots i_7}\hat{\mathcal{K}}^{(2)}_{i_8 i_9} \\
&+\frac{1}{2^3\cdot 5!}(i_{\hat{k}}\widetilde{\hat{C}^{(6)}})_{i_1\cdots i_5}(\hat{\mathcal{K}}^{(2)})^2_{i_6\cdots i_9} + \frac{1}{2\cdot(3!)^2}\tilde{\hat{C}}_{i_1i_2i_3}\hat{\mathcal{K}}^{(2)}_{i_4i_5}\{(\partial\hat{b})^2 - \frac{1}{4}(\widetilde{i_{\hat{k}}\hat{C}})\partial\hat{b} + \frac{1}{8}\widetilde{(i_{\hat{k}}\hat{C})}^2\}_{i_6\cdots i_9} \\
&+\frac{1}{2\cdot 4!}\hat{A}_{i_1}\hat{\mathcal{K}}^{(2)}_{i_2i_3}\{(\partial\hat{b})^3 + \frac{1}{2}(\partial\hat{b})^2(\widetilde{i_{\hat{k}}\hat{C}}) + \frac{1}{4}(\partial\hat{b})\widetilde{(i_{\hat{k}}\hat{C})}^2 + \frac{1}{8}\widetilde{(i_{\hat{k}}\hat{C})}^3\}_{i_4\cdots i_9} \\
&+\frac{m}{5!}\hat{b}_{i_1}(\partial\hat{b})^4_{i_2\cdots i_9} + \frac{1}{8!}\partial_{i_1}\hat{\omega}^{(8)}_{i_2\cdots i_9}]
\end{aligned} \tag{2.21}$$

where $\widetilde{\hat{T}^{(r)}}_{i_1\cdots i_r} \equiv \hat{T}^{(r)}_{\hat{\mu}_1\cdots\hat{\mu}_r}D_{i_1}\hat{X}^{\hat{\mu}_1}\cdots D_{i_r}\hat{X}^{\hat{\mu}_r}$ for a target-space r-form field $\hat{T}^{(r)}_{\hat{\mu}_1\cdots\hat{\mu}_r}$. $\hat{b}_i$ describes the flux of an M-2-brane wrapped around the isometry direction[29], whose massive gauge transformation is determined by the requirement of the invariance of its modified field strength $\hat{\mathcal{K}}^{(2)}_{ij} = 2\partial_{[i}\hat{b}_{j]} - D_i\hat{X}^{\hat{\mu}}D_j\hat{X}^{\hat{\nu}}(i_{\hat{k}}\hat{C})_{\hat{\mu}\hat{\nu}}$ (i.e. $\delta\hat{b}_i = \hat{\lambda}_i$). We note that S^{WZ}_{M9} is exactly gauge invariant if the worldvolume 8-form $\hat{\omega}^{(8)}$ is transformed as

$$\delta\hat{\omega}^{(8)} = 8!\{\frac{1}{6!}\partial(i_{\hat{k}}\hat{\Omega}^{(7)})\hat{b} + \frac{1}{2\cdot 4!}\partial(i_{\hat{k}}\hat{\chi}^{(5)})\hat{b}\partial\hat{b} + \frac{1}{2\cdot 3!}\partial\hat{\chi}^{(2)}\hat{b}(\partial\hat{b})^2 - \frac{4}{5!}m\hat{\lambda}\hat{b}(\partial\hat{b})^3\}. \tag{2.22}$$

Actually, the expression of the action using the covariant derivative DX is not so convenient to discuss dimensional reductions of the action S^{WZ}_{M9}. So, we rewrite the action into the expression without DX using the two identities:

$$(i_{\hat{k}}\widetilde{\hat{T}^{(r)}})_{i_1\cdots i_{r-1}} = (i_{\hat{k}}\hat{T}^{(r)})_{i_1\cdots i_{r-1}}, \quad \widetilde{\hat{T}^{(r)}}_{i_1\cdots i_r} = \hat{T}^{(r)}_{i_1\cdots i_r} - r\cdot\hat{A}_{[i_1}(i_{\hat{k}}\hat{T}^{(r)})_{i_2\cdots i_r]}, \tag{2.23}$$

where $\hat{T}^{(r)}_{i_1\cdots i_r} \equiv \hat{T}^{(r)}_{\hat{\mu}_1\cdots\hat{\mu}_r}\partial_{i_1}\hat{X}^{\hat{\mu}_1}\cdots\partial_{i_1}\hat{X}^{\hat{\mu}_1}$. We note that $D_i\hat{X}^{\hat{\mu}} = \partial_i\hat{X}^{\hat{\mu}}$ for $\hat{\mu} \neq z$ and $D_i\hat{X}^z = -\hat{A}_i$. Then, the WZ action (2.21) is written as the slightly simple form:

$$\begin{aligned}
S^{WZ}_{M9} \quad &= \quad \int d^9\xi\hat{\epsilon}^{i_1\cdots i_9}[\frac{1}{9!}(i_{\hat{k}}\hat{C}^{(10)})_{i_1\cdots i_9} + \frac{1}{2\cdot 7!}(i_{\hat{k}}\hat{N}^{(8)})_{i_1\cdots i_7}\hat{\mathcal{K}}'^{(2)}_{i_8 i_9} \\
&+\frac{1}{2^3\cdot 5!}(i_{\hat{k}}\hat{C}^{(6)})_{i_1\cdots i_5}(\hat{\mathcal{K}}'^{(2)})^2_{i_6\cdots i_9} + \frac{1}{2\cdot(3!)^2}\hat{C}_{i_1i_2i_3}\hat{\mathcal{K}}'^{(2)}_{i_4i_5}\{(\partial\hat{b})^2 - \frac{1}{4}(i_{\hat{k}}\hat{C})\partial\hat{b} + \frac{1}{8}(i_{\hat{k}}\hat{C})^2\}_{i_6\cdots i_9} \\
&+\frac{1}{2^4\cdot 4!}\hat{A}_{i_1}(\hat{\mathcal{K}}'^{(2)})^4_{i_2\cdots i_9} + \frac{m}{5!}\hat{b}_{i_1}(\partial\hat{b})^4_{i_2\cdots i_9} + \frac{1}{8!}\partial_{i_1}\hat{\omega}^{(8)}_{i_2\cdots i_9}]
\end{aligned} \tag{2.24}$$

where $\hat{\mathcal{K}}'^{(2)}_{ij} = 2\partial_{[i}\hat{b}_{j]} - \partial_i\hat{X}^{\hat{\mu}}\partial_j\hat{X}^{\hat{\nu}}(i_{\hat{k}}\hat{C})_{\hat{\mu}\hat{\nu}}$. We use this form to discuss dimensional reductions of S^{WZ}_{M9}.

3 Dimensional reductions of the M-9-brane WZ action

Now we discuss the dimensional reductions of S^{WZ}_{M9}. Since the purpose of this work is to examine the consistency, the important point is that *precisely the same forms of the actions are derived.* Thus, in each case of the reductions, we present explicit correspondence of the terms in S^{WZ}_{M9} with those in the WZ action of the other brane.

3. 1 The dimensional reduction along the isometry direction

First, we discuss the dimensional reduction of S^{WZ}_{M9} along the isometry direction and show that WZ action of the D-8-brane *with no isometry direction* is derived. In this case we split the coordinates $\hat{x}^{\hat{\mu}}$ for $\hat{\mu} = 0, 1, \cdots, 9, z$ into (x^μ, z) $(\mu = 0, 1, \cdots, 9)$. Since one of the worldvolume directions of the M-9-brane is considered to be wrapped around the isometry direction, all we have to do is to rewrite S^{WZ}_{M9} in terms of 10D fields.

The relations of original 11D target-space fields to the 10D ones can be taken as the familiar form[17]

$$\left\{\begin{array}{ll} \hat{g}_{\mu\nu} = & e^{-2\phi/3} g_{\mu\nu} - e^{4\phi/3} C^{(1)}_\mu C^{(1)}_\nu, \\ \hat{g}_{\mu z} = & (i_{\hat{k}}\hat{g})_\mu = -e^{4\phi/3} C^{(1)}_\mu \\ \hat{g}_{zz} = & -e^{4\phi/3} \end{array}\right. \qquad \left\{\begin{array}{ll} \hat{C}_{\mu\nu\rho} = & C^{(3)}_{\mu\nu\rho} \\ \hat{C}_{\mu\nu z} = & (i_{\hat{k}}\hat{C})_{\mu\nu} = B_{\mu\nu}. \end{array}\right. \tag{3.1}$$

where B is the NSNS 2-form and $C^{(1)}$ and $C^{(3)}$ are the 10D RR 1-form and 3-form potentials (we denote 10D RR r-forms as $C^{(r)}$). By using these relations, the massive 11D action (2.6) is shown to give the bosonic part of the action of 10D massive IIA supergravity given in ref.[12] on the dimensional reduction[17]. In particular, the third term of (2.6) gives rise to the cosmological term $-m^2/2$, and the kinetic term of $\hat{C}$ gives a mass term of $B_{\mu\nu}$.

The dimensional reductions of the other target-space gauge fields are given as[17][20]

$$\left\{\begin{array}{ll} \hat{C}^{(6)}_{\mu_1\cdots\mu_6} = & -\tilde{B}^{(6)}_{\mu_1\cdots\mu_6}, \\ \hat{C}^{(6)}_{\mu_1\cdots\mu_5 z} = & (i_{\hat{k}}\hat{C}^{(6)})_{\mu_1\cdots\mu_5} = C^{(5)}_{\mu_1\cdots\mu_5} - 5C^{(3)}_{[\mu_1\mu_2\mu_3} B_{\mu_4\mu_5]}, \end{array}\right. \tag{3.2}$$

$$(i_{\hat{k}}\hat{N}^{(8)})_{\mu_1\cdots\mu_7} = C^{(7)}_{\mu_1\cdots\mu_7} - 7\cdot 5 C^{(3)}_{[\mu_1\mu_2\mu_3} B_{\mu_4\mu_5} B_{\mu_6\mu_7]} \tag{3.3}$$

$$(i_{\hat{k}}\hat{C}^{(10)})_{\mu_1\cdots\mu_9} = C^{(9)}_{\mu_1\cdots\mu_9} \tag{3.4}$$

where $\tilde{B}^{(6)}$ is the NSNS 6-form gauge field, the dual of B. The relations of worldvolume fields are that $\hat{b}_i$ corresponds to the BI field on the D-8-brane V_i, and $\hat{\omega}^{(8)}$ corresponds to an 8-form $\omega^{(8)}$.

Then, dimensionally reducing S^{WZ}_{M9} along z, we have the following action

$$S'^{WZ} = \int d^9\xi \epsilon^{i_1\cdots i_9} [\sum_{r=1,3,5,7,9} \{\frac{1}{r!q!2^q} C^{(r)} \mathcal{F}^q\} + \frac{m}{5!} V(\partial V)^4 + \frac{1}{8!}\partial\omega^{(8)}]_{i_1\cdots i_9} \tag{3.5}$$

where $\mathcal{F}_{ij} \equiv 2\partial_{[i}V_{j]} - B_{ij}$, $q = (9-r)/2$ and $T^{(r)}_{i_1\cdots i_r} \equiv T^{(r)}_{\mu_1\cdots\mu_r}\partial_{i_1}X^{\mu_1}\cdots\partial_{i_r}X^{\mu_r}$ for a field $T^{(r)}_{\mu_1\cdots\mu_r}$. This is exactly the bosonic part of the D-8-brane WZ action in a massive IIA background[27][28]. Thus, we can say that *this reduction of the M-9-brane is consistent with the relations of the branes given in Fig.1.*

The explicit correspondence of the terms in S^{WZ}_{M9} with those in S^{WZ}_{D8} is given as follows: By using (3.2), (3.3) and (3.4), the first three terms of (2.24) give the terms including $C^{(9)}$, $C^{(7)}$ and $C^{(5)}$ in the action (3.5), in addition to the contribution

$$\frac{1}{2^3(3!)^2}\{C^{(3)}\mathcal{F}(\frac{B^2}{2}-3B\partial V)\}_{i_1\cdots i_9}, \tag{3.6}$$

since $\hat{\mathcal{K}}'^{(2)}_{ij}$ gives $\mathcal{F}_{ij}$. This term (3.6), combined with the contribution of the forth term in (2.24) gives the term including $C^{(3)}$ in the action (3.5). Finally, the fifth term of (2.24) (including $\hat{A}$) gives the term including $C^{(1)}$ in the action (3.5) since it holds $\hat{A}_i = C^{(1)}_\mu \partial_i X^\mu$.

3. 2 The dimensional reduction along a standard worldvolume direction

Next, we show that on this double dimensional reduction, the action S''^{WZ} obtained from S^{WZ}_{M9} can be identified with the WZ action of the KK-8A brane S^{WZ}_{KK8A} presented in ref.[25], which we consider to be a D-8-brane with an isometry direction due to some special background.

The outline of this identification is as follows: First, we derive the action S''^{WZ} and discuss the difference of the obtained action S''^{WZ} from S^{WZ}_{KK8A}. Then, the two looks certainly different from each other at a glance. However, there is a target-space field appearing in each of the action whose relation to each other is not known yet; a 9-form potential $C'^{(9)}$ appearing in S''^{WZ} which originates from the 11D 10-form $\hat{C}^{(10)}$, and a 9-form $N^{(9)}$ appearing in S^{WZ}_{KK8A}[25] which comes from the 10D IIB 8-form via T-duality. The relation of these two 9-forms has not been discussed, while the other fields we use are completely the same as those in ref.[25].* Thus, we can say that *the two actions are equivalent if the difference of the two can be cancelled only by setting the field redefinition relation of the two 9-forms.* In addition, we can check the consistency of the redefinition relation by discussing the gauge transformations of both sides of the relation. We will demonstrate these in the following.

In this case we denote the worldvolume indices of the M-9-brane by $\hat{i}$, and split the coordinates $\hat{x}^{\hat{\mu}}$ into (x^μ, x^8) $(\mu = 0, 1, \cdots, 7, 9, z)$ and $\xi^{\hat{i}}$ into (ξ^i, ξ^8) $(i = 0, 1, \cdots, 7)$. Then, we fix the coordinates so that $X^8(\xi) = \xi^8$ and consider the double dimensional reduction along $X^8 = \xi^8$. Please remember that on this reduction, one of the indices i_r $(r = 1, \cdots, 9)$ in the M-9-brane WZ action (2.24) always takes the value of 8, and that it holds $\partial_8 \hat{X}^{\hat{\mu}} = \delta^{\hat{\mu}}_8$.

* The distinctions of the fields are made on the basis of their gauge transformation properties and the 11D origins of the fields.

As for the target-space fields, the relations of $\hat{g}_{\hat{\mu}\hat{\nu}}$ and $\hat{C}$ with the 10D fields are the same as (3.1) except for the replacement of the indices $z \to 8$, but the obtained theory is not the usual massive IIA SUGRA in ref.[13][12] because there remains an isometry direction. That is, this is another massive extension of standard (massless) IIA SUGRA. However, based on the analyses on dimensional reductions of the massive 11D SUGRA to 9-dimensional SUGRAs, the authors of ref.[23] argue that the obtained another massive IIA SUGRA is related to the usual massive IIA one by a rotation in internal space. We believe their argument and consider the field content appearing in this massive SUGRA to be essentially the same as that in the usual massive IIA one. We note that the 10D Killing vector k^μ is defined as $k^\mu \equiv \hat{k}^\mu$ ($\hat{k}^8 = 0$).

The dimensional reductions of the dual fields are as follows: That of $\hat{C}^{(6)}$ is again the same as (3.2) except for the replacement $z \to 8$. On the other hand, the dimensional reduction of $i_{\hat{k}}\hat{N}^{(8)}$ is given by[29][25]

$$\left\{ \begin{array}{rcl} (i_{\hat{k}}\hat{N}^{(8)})_{\mu_1\cdots\mu_7} &=& -(i_k N^{(8)})_{\mu_1\cdots\mu_7}, \\ (i_{\hat{k}}\hat{N}^{(8)})_{\mu_1\cdots\mu_6 y} &=& (i_k N^{(7)})_{\mu_1\cdots\mu_6} \end{array} \right. \tag{3.7}$$

where $i_k N^{(8)}$ is the 8-form dual of the "scalar field" $(i_k C^{(1)})$ while $i_k N^{(7)}$ is the 7-form dual of the 1-form field $k_\mu = (i_k g)_\mu$. The duality relations they satisfy are derived from the second equation of (2.11). The brane which couples to $i_k N^{(8)}$ is called "KK-6A brane"[23][24][25], the solution corresponding to which is shown to be identified with the D-6-brane solution via a coordinate transformation[25]. The brane which couples to $i_k N^{(7)}$ is a IIA KK-monopole. We note that $\hat{N}^{(8)}$ gives $N^{(7)}$ but not $C^{(7)}$ unlike (3.3). This is possible because the definition of $i_{\hat{k}}\hat{N}^{(8)}$ itself, the 11D origin of $N^{(7)}$, depends on $\hat{k}$. (see eq.(2.11)).

The dimensional reduction of $(i_{\hat{k}}\hat{C}^{(10)})$ is written as

$$\left\{ \begin{array}{rcl} (i_{\hat{k}}\hat{C}^{(10)})_{\mu_1\cdots\mu_9} &=& -(i_k B^{(10)})_{\mu_1\cdots\mu_9} \\ (i_{\hat{k}}\hat{C}^{(10)})_{\mu_1\cdots\mu_8 y} &=& -(i_k C'^{(9)})_{\mu_1\cdots\mu_8} \end{array} \right. \tag{3.8}$$

where $B^{(10)}$ is a NSNS 10-form to which the NS-9A brane couples[10]. ($B^{(10)}$ does not appear in S^{WZ}_{KK8A} since in this reduction one of the indices of $i_{\hat{k}}\hat{C}^{(10)}$ arising in the action (2.24) takes the value of 8.) $C'^{(9)}$ is considered to be a kind of RR 9-form potential, to which the KK-8A brane is expected to couple minimally. The difference between this 9-form $C'^{(9)}$ and the usual $C^{(9)}$ coming from the existence of the isometry appears in the duality relations they satisfy. For a constant mass background $\hat{M} = m$, they are given respectively as

$$m|k|^4 e^{-4\phi} = *(dC'^{(9)} + \cdots) \tag{3.9}$$

$$m = *(dC^{(9)} + \cdots) \tag{3.10}$$

where $\cdots$ are the parts not essential to our discussion. The gauge transformations of $B^{(10)}$ and $C'^{(9)}$ are obtained from (2.19). We note that since $\hat{C}^{(10)}$ has no dynamical degrees of freedom, so do $B^{(10)}$ and $C'^{(9)}$.

As for the worldvolume gauge fields, we split $\hat{b}_{\hat{i}}$ into $(\omega^{(1)}_i, \omega^{(0)})$, as ref.[25], and define a worldvolume 7-form as the dimensional reduction of $\hat{\omega}^{(8)}$: $\omega^{(7)} \equiv \hat{\omega}^{(8)}_{i_1\cdots i_7 8}$. (We note that $\hat{\omega}^{(8)}$ with no index of 8 does not appear in the action.) By using (3.1), the dimensional reduction of $\hat{A}_{\hat{i}}$ $(= -|\hat{k}|^{-2}\partial_{\hat{i}}\hat{X}^{\hat{\mu}}\hat{k}_{\hat{\mu}})$ is written as

$$
\begin{aligned}
\hat{A}_i &= \{1 + e^{2\phi}|k|^{-2}(i_k C^{(1)})^2\}^{-1}\{A_i + e^{2\phi}|k|^{-2}(i_k C^{(1)})C^{(1)}_i\} \\
&= A_i + \{1 + e^{2\phi}|k|^{-2}(i_k C^{(1)})^2\}^{-1} e^{2\phi}|k|^{-2}(i_k C^{(1)}) D_i X^\mu C^{(1)}_\mu \qquad (3.11) \\
\hat{A}_8 &= \{1 + e^{2\phi}|k|^{-2}(i_k C^{(1)})^2\}^{-1} e^{2\phi}|k|^{-2}(i_k C^{(1)}) \qquad (3.12)
\end{aligned}
$$

where $A_i \equiv -|k|^{-2}\partial_i X^\mu k_\mu$ and $D_i X^\mu \equiv \partial_i X^\mu - A_i k^\mu$ is the worldvolume covariant derivative on the KK-8A brane[29]. For later use, we define the "field strengths" of $\omega^{(1)}$ and $\omega^{(0)}$ as

$$
\begin{aligned}
\mathcal{K}'^{(2)}_{ij} &\equiv \hat{\mathcal{K}}^{(2)}_{ij} = 2\partial_{[i}\omega^{(1)}_{j]} - (i_k C^{(3)})_{ij} \qquad (3.13) \\
\mathcal{K}^{(1)}_i &\equiv \hat{\mathcal{K}}^{(2)}_{i8} = \partial_i \omega^{(0)} + (i_k B)_i. \qquad (3.14)
\end{aligned}
$$

Now, let us discuss the difference between S^{WZ}_{KK8A} and the action S''^{WZ} which will be obtained from S^{WZ}_{M9}. Since the action S''^{WZ} and S^{WZ}_{KK8A} are both too complicated to deal with at once, we divide the contribution of S^{WZ}_{M9} into 4 parts ; the part (I): terms including p-form potentials with $p > 3$, the part (II): those including $C^{(3)}_{ijk}$ (which does not contain k^μ), the part (III): those including B_{ij} (which does not contain k^μ), and the part (IV): the rest part. We will compare each part of S''^{WZ} separately with that of S^{WZ}_{KK8A}, which in our notation takes the form[25]:†

$$
\begin{aligned}
S^{WZ}_{KK8A} &= \int d^8\xi\, \epsilon^{(8)} \cdot [-\frac{1}{8!}(i_k N^{(9)}) - \frac{1}{7!}(i_k N^{(8)})\partial\omega^{(0)} \\
&+ \frac{1}{2\cdot 6!}(i_k N^{(7)})\{\mathcal{K}'^{(2)} - 2(i_k B^{(2)})DXC^{(1)}\} - \frac{1}{2\cdot 5!}(i_k B^{(6)})\mathcal{K}^{(1)}\mathcal{K}^{(2)} \\
&- \frac{1}{2^3\cdot 4!}(i_k C^{(5)})\{\mathcal{K}^{(2)} + 4\mathcal{K}^{(1)}(DXC^{(1)})\}\mathcal{K}^{(2)} \\
&- \frac{1}{24\cdot 3!}\widetilde{C^{(3)}}\{2((i_k C^{(3)})(i_k B) - 3\mathcal{K}^{(1)}\partial\omega^{(1)})\mathcal{K}^{(2)} - (i_k C^{(3)})^2\partial\omega^{(0)}\} \\
&+ \frac{1}{12\cdot 3!}\{2C^{(3)}(i_k B) + 3(i_k C^{(3)})B\}(i_k C^{(3)})(DXC^{(1)})\partial\omega^{(0)} \\
&- \frac{1}{48}\widetilde{B}\{(i_k C^{(3)})^2\mathcal{K}^{(2)} - 4(\partial\omega^{(1)})^3\} + \frac{1}{3!}A\partial\omega^{(0)}(\partial\omega^{(1)})^3
\end{aligned}
$$

†In ref.[25], the factor $1/2^6$ of the first term of the last line is omitted. Note that the notation in ref.[25] is changed as $C^{(3)} \to -C^{(3)}$, $B \to -B$ and $A_i \to -A_i$.

$$+\quad \frac{1}{2^6\cdot 3!}\frac{e^{2\phi}|k|^{-2}(i_kC^{(1)})}{1+e^{2\phi}|k|^{-2}(i_kC^{(1)})^2}(\mathcal{K}^{(2)})^4+\frac{1}{7!}\partial\omega^{(7)}], \tag{3.15}$$

where $\epsilon\cdot T\equiv\epsilon^{i_1\cdots i_8}T_{i_1\cdots i_8}$ for (products of) fields $T_{i_1\cdots i_8}$ and $\mathcal{K}^{(2)}=\mathcal{K}'^{(2)}-2\mathcal{K}^{(1)}(DXC^{(1)})$. $N^{(9)}$ is a 9-form gauge field introduced in ref.[25], which minimally couples to the KK-8A brane. We note that it holds $D_iX^\mu C^{(1)}_\mu=\partial_iX^\mu[C^{(1)}_\mu+|k|^{-2}k_\mu(i_kC^{(1)})]$.

First, we consider the part (I). That of $\mathcal{L}''^{WZ}$ comes from the first three terms of the M-9-brane WZ action (2.24), which give the following contribution:

$$\begin{aligned}
&\hat\epsilon\cdot[\frac{1}{9!}(i_{\hat k}\hat C^{(10)})+\frac{1}{2\cdot 7!}\{(i_{\hat k}\hat N^{(8)})\hat{\mathcal{K}'}^{(2)}\}+\frac{1}{2^3\cdot 5!}(i_{\hat k}\hat C^{(6)})(\hat{\mathcal{K}'}^{(2)})^2]\,|_{Dimensional\ Reduction}\\
=\;&\epsilon^{(8)}\cdot[-\frac{1}{8!}(i_kC'^{(9)})-\frac{1}{7!}(i_kN^{(8)})\mathcal{K}^{(1)}+\frac{1}{2\cdot 6!}(i_kN^{(7)})\mathcal{K}'^{(2)}-\frac{1}{2\cdot 5!}(i_kB^{(6)})\mathcal{K}'^{(2)}\mathcal{K}^{(1)}\\
&-\frac{1}{2^3\cdot 4!}(i_kC^{(5)})(\mathcal{K}'^{(2)})^2+\frac{1}{2^3\cdot 4!}\{3(i_kC^{(3)})B+2C^{(3)}(i_kB)\}(\mathcal{K}'^{(2)})^2]
\end{aligned}\tag{3.16}$$

where $\hat\epsilon\cdot\hat T\equiv\hat\epsilon^{\hat i_1\cdots\hat i_9}\hat T_{\hat i_1\cdots\hat i_9}$ and we identify $\epsilon^{(8)i_1\cdots i_8}\equiv\hat\epsilon^{i_1\cdots i_8 8}$. So, the first five terms of the right hand side of (3.16) is the part (I), which we denote as $\mathcal{L}''^{WZ}|_{(I)}$. On the other hand, the part (I) of $\mathcal{L}^{WZ}_{KK8A}$ corresponds to the upper three lines of (3.15). Subtracting $\mathcal{L}''^{WZ}|_{(I)}$ from the upper three lines of (3.15), we have the difference of the two‡

$$\begin{aligned}
\{\mathcal{L}^{WZ}_{KK8A}-\mathcal{L}''^{WZ}\}|_{(I)}=\frac{1}{8!}\epsilon^{(8)}\cdot[&(i_kC'^{(9)})-(i_kN^{(9)})+8(i_kN^{(8)})(i_kB)\\
&-56(i_kN^{(7)})(i_kB)\{C^{(1)}+|k|^{-2}k_\mu\partial X^\mu(i_kC^{(1)})].
\end{aligned}\tag{3.17}$$

The right hand side of (3.17) consists only of some products of target-space fields, except for the factor ∂X. Thus, the difference can be absorbed in the field redefinition relation of $C'^{(9)}$ and $N^{(9)}$.

Let us next consider the part (II) of $\mathcal{L}''^{WZ}$, which comes from the forth term of (2.24) and the last term of (3.16), as

$$\begin{aligned}
\mathcal{L}''^{WZ}|_{(II)}=\frac{1}{2^3\cdot 3^2}\epsilon^{(8)}\cdot[&C^{(3)}\{6(\partial\omega^{(1)})^2\partial\omega^{(0)}+6(i_kB)(\partial\omega^{(1)})^2-3(i_kC^{(3)})\partial\omega^{(1)}\partial\omega^{(0)}\\
&-5(i_kB)(i_kC^{(3)})\partial\omega^{(1)}+\frac{1}{2}(i_kC^{(3)})^2\partial\omega^{(0)}+\frac{3}{2}(i_kB)(i_kC^{(3)})^2\}].
\end{aligned}\tag{3.18}$$

The part (II) of $\mathcal{L}^{WZ}_{KK8A}$ comes from the forth line and the first term of the fifth line in (3.15). The difference of the two is

$$\{\mathcal{L}^{WZ}_{KK8A}-\mathcal{L}''^{WZ}\}|_{(II)}\sim-\frac{1}{2^4\cdot 3^2}\epsilon^{(8)}\cdot[C^{(3)}(i_kC^{(3)})^2(i_kB)], \tag{3.19}$$

‡In deriving (3.17), the following identities are useful: $\mathcal{K}^{(1)}(\mathcal{K}^{(2)}-\mathcal{K}'^{(2)})\propto\mathcal{K}^{(1)}\wedge\mathcal{K}^{(1)}=0$ and $\{\mathcal{K}^{(2)}+4\mathcal{K}^{(1)}DXC^{(1)}\}\mathcal{K}^{(2)}=(\mathcal{K}'^{(2)})^2$.

which also can be absorbed in the redefinition of the 9-forms. The part (III) is also discussed in the same way. The part (III) of $\mathcal{L}''^{WZ}$ comes from the forth term of (2.24) and the sixth term of (3.16):

$$\mathcal{L}''^{WZ}|_{(III)} = \frac{1}{2^4 \cdot 3!}\epsilon^{(8)} \cdot [B\{8(\partial\omega^{(1)})^3 - 4(i_k C^{(3)})^2 \partial\omega^{(1)} + (i_k C^{(3)})^3\}]. \tag{3.20}$$

The part (III) of S^{WZ}_{KK8A} comes from the second term of the fifth line and the first term of the sixth line in (3.15). The difference of this part is

$$\begin{aligned}\{\mathcal{L}^{WZ}_{KK8A} - \mathcal{L}''^{WZ}\}|_{(III)} &= \epsilon^{(8)} \cdot \frac{1}{2^4 \cdot 3!} B[(i_k C^{(3)})^3 \\ &+ 4(i_k C^{(3)})^2 (i_k B)\{C^{(1)} + |k|^{-2} k_\mu \partial X^\mu (i_k C^{(1)})\}]\end{aligned} \tag{3.21}$$

which can also be absorbed in the same way.

Finally, we consider the part (IV), the rest part of the actions. We can easily see that the last term of (2.24) gives the last term of (3.15). The sixth term $\frac{m}{5!}\hat{b}_{i_1}(\partial\hat{b})^4$ in (2.24) is the "massive part", i.e. the part which arises when the background is a massive one. In fact, the massive part of the KK-8A brane action has not been discussed in ref.[25]. This means that (3.15) is not the action describing the KK-8A brane in the most generic background. The purpose of this subsection is to examine the correspondence of the *same part* of the two actions. Thus, in this subsection we set the mass parameter to be zero ($m = 0$) and concentrate our discussion on the massless part of the two actions. We will argue the validity of this discussion under the setting $m = 0$ and discuss the massive part of the action in the final section.

The rest of the part (IV) of $\mathcal{L}''^{WZ}$ comes from the fifth terms of (2.24):

$$\begin{aligned}\mathcal{L}''^{WZ}|_{(IV)} &= \frac{1}{2 \cdot 4!}\epsilon^{(8)} \cdot A[\mathcal{K}^{(1)}(\mathcal{K}'^{(2)})^3] \\ &+ \frac{1}{2^4 \cdot 4!}\frac{e^{2\phi}|k|^{-2}(i_k C^{(1)})}{1 + e^{2\phi}|k|^{-2}(i_k C^{(1)})^2}\epsilon^{(8)} \cdot [(\mathcal{K}'^{(2)})^4 + 8(DXC^{(1)})\mathcal{K}^{(1)}(\mathcal{K}'^{(2)})^3].\end{aligned} \tag{3.22}$$

To derive this, we use the second expression of (3.11). The second term of (3.22) exactly reproduces the last line of (3.15). The rest of the contribution of $\mathcal{L}^{WZ}_{KK8A}$ comes from the forth and the sixth lines of (3.15) since $\widetilde{T^{(r)}} = T^{(r)} - r \cdot A(i_k T^{(r)})$. After a little calculation, we obtain the difference of this part as

$$\{\mathcal{L}^{WZ}_{KK8A} - \mathcal{L}''^{WZ}\}|_{(IV)} = \frac{1}{48}\epsilon^{(8)} \cdot [|k|^{-2} k_\mu \partial X^\mu (i_k B)(i_k C^{(3)})^3]. \tag{3.23}$$

which can also be absorbed in the same way as the above. (We note that $A_i = -|k|^{-2} k_\mu \partial_i X^\mu$.)

Therefore, putting together all the differences presented above, we can see that the two actions are equivalent if the following field redefinition relation of the 9-forms holds:

$$(i_k C'^{(9)}) = (i_k N^{(9)}) - 8(i_k N^{(8)})(i_k B) + 56(i_k N^{(7)})(i_k B)\{C^{(1)} + |k|^{-2} k_{(\mu)}(i_k C^{(3)})\}$$

$$
\begin{aligned}
&+\quad 280C^{(3)}(i_kC^{(3)})^2(i_kB)\\
&-\quad 420B[(i_kC^{(3)})^3+4(i_kC^{(3)})^2(i_kB)\{C^{(1)}+|k|^{-2}k_{(\mu)}(i_kC^{(1)})\}]\\
&-\quad 840|k|^{-2}k_{(\mu)}(i_kC^{(3)})^3(i_kB).
\end{aligned}
\tag{3.24}
$$

Thus, all we have to do now is only to show that (3.24) really holds. Actually, correspondence of gauge transformations of the fields is the only criterion in this theory to discuss the consistency the field redefinition relation; no other ingredients to judge the consistency, such as supersymmetry, have not been discussed in this theory.§ To be concrete, we prove that the gauge transformations of both sides of (3.24) agree with each other.

First, we discuss the gauge transformation of the right hand side (r.h.s) of (3.24). In the massless case $m=0$, the gauge transformations of fields are defined as[17][30][25]

$$
\begin{aligned}
\delta(i_kN^{(9)}) &= 8\partial(i_k\Omega^{(8)})-168\partial(i_k\Omega^{(6)})[i_kC^{(3)}+2(i_kB)\{C^{(1)}+|k|^{-2}k_{(\mu)}(i_kC^{(1)})\}]\\
&+\quad 840\partial(i_k\Lambda^{(4)})(i_kC^{(3)})[i_kC^{(3)}+4(i_kB)\{C^{(1)}+|k|^{-2}k_{(\mu)}(i_kC^{(1)})\}]\\
&+\quad 2520\partial(i_k\Lambda^{(2)})(i_kC^{(3)})[(i_kC^{(3)})\{B-2|k|^{-2}k_{(\mu)}(i_kB)\}\\
&\qquad\qquad +4B(i_kB)\{C^{(1)}+|k|^{-2}k_{(\mu)}(i_kC^{(1)})\}]\\
&-\quad 8\partial(i_k\Lambda^{(1)})[i_kN^{(8)}+7(i_kN^{(7)})\{C^{(1)}+|k|^{-2}k_{(\mu)}(i_kC^{(1)})\}\\
&\qquad\qquad -35(i_kC^{(3)})^2\{C^{(3)}+3|k|^{-2}k_{(\mu)}(i_kC^{(3)})\}\\
&-\quad 70(i_kC^{(3)})\{2C^{(3)}(i_kB)+3(i_kC^{(3)})B\}\{C^{(1)}+|k|^{-2}k_{(\mu)}(i_kC^{(1)})\}]
\end{aligned}
\tag{3.25}
$$

$$
\begin{aligned}
\delta(i_kN^{(8)}) &= 7\partial(i_k\Omega^{(7)})+105\partial(i_k\Lambda^{(5)})(i_kC^{(3)})-210\partial\Lambda^{(2)}(i_kC^{(3)})^2\\
&+\quad 140\partial(i_k\Lambda^{(2)})C^{(3)}(i_kC^{(3)})-7\partial\Lambda^{(0)}(i_kN^{(7)})
\end{aligned}
\tag{3.26}
$$

$$
\begin{aligned}
\delta N^{(7)} &= 6\partial(i_k\Omega^{(6)})+30\partial(i_k\Lambda^{(5)})(i_kB)-60\partial(i_k\Lambda^{(4)})(i_kC^{(3)})\\
&-\quad 120\partial\Lambda^{(2)}(i_kC^{(3)})(i_kB)+20\partial(i_k\Lambda^{(2)})\{2C^{(3)}(i_kB)-3(i_kC^{(3)})B\}\\
&\quad -20\partial(i_k\Lambda^{(1)})C^{(3)}(i_kC^{(3)})
\end{aligned}
\tag{3.27}
$$

$$\delta\tilde{B}^{(6)} = 6\partial\Lambda^{(5)}-30\partial\Lambda^{(2)}C^{(3)}+6\partial\Lambda^{(0)}\{C^{(5)}-5C^{(3)}B\} \tag{3.28}$$

$$\delta C^{(5)} = 5\partial\Lambda^{(4)}+30\partial\Lambda^{(2)}B+15\partial\Lambda^{(0)}BB \tag{3.29}$$

$$\delta C^{(3)} = 3\partial\Lambda^{(2)}+3\partial\Lambda^{(0)}B \tag{3.30}$$

$$\delta B = 2\partial\Lambda^{(1)} \tag{3.31}$$

$$\delta C^{(1)} = \partial\Lambda^{(0)} \tag{3.32}$$

where $\Omega^{(r)}$ and $\Lambda^{(r)}$ are the r-form gauge parameters associated with the (r+1)-form

§In addition, although the 9-form $N^{(9)}$ is introduced in ref.[25] as a field to couple to the KK-8A brane, the field strength of $N^{(9)}$ or the duality relation which it should satisfy have not been given. Thus, there is no other way to discuss the consistency of (3.24). Conversely, we are able to determine the field strength of $N^{(9)}$ through (3.24), by deriving the field strength of $C^{'(9)}$ from the field strength of $\hat{C}^{(10)}$ via dimensional reduction.

gauge potentials, respectively.[¶] We note that these parameters, except for the $\Omega^{(8)}$ which is associated with $N^{(9)}$, are related to those in 11D as[‖]

$$\begin{cases} (i_{\hat{k}}\hat{\Omega}^{(7)})_{\mu_1\cdots\mu_6} = & -(i_k\Omega^{(7)})_{\mu_1\cdots\mu_6} \\ (i_{\hat{k}}\hat{\Omega}^{(7)})_{\mu_1\cdots\mu_5 8} = & (i_k\Omega^{(6)})_{\mu_1\cdots\mu_5} \end{cases} \quad \begin{cases} (i_{\hat{k}}\hat{\chi}^{(5)})_{\mu_1\cdots\mu_4} = & -(i_k\Lambda^{(5)})_{\mu_1\cdots\mu_4} \\ (i_{\hat{k}}\hat{\chi}^{(5)})_{\mu_1\cdots\mu_3 8} = & -(i_k\Lambda^{(4)})_{\mu_1\cdots\mu_3} \end{cases}$$
$$\begin{cases} \hat{\chi}_{\mu\nu} = & \Lambda^{(2)}_{\mu\nu} \\ \hat{\chi}_{\mu 8} = & \Lambda^{(1)}_{\mu}. \end{cases} \tag{3.33}$$

After a bit lengthy calculation, the *massless part* of the transformation of the r.h.s of (3.24) is written as

$$\begin{aligned} \delta(\text{r.h.s. of (3.24)}) &= 8\partial(i_k\Omega^{(8)}) - 56(i_k\Omega^{(7)})(i_kB) - 168(i_k\Omega^{(6)})(i_kC^{(3)}) \\ &- 840\partial(i_k\Lambda^{(5)})(i_kC^{(3)})(i_kB) + 840\partial(i_k\Lambda^{(4)})(i_kC^{(3)})^2 \\ &+ 2520\partial\Lambda^{(2)}(i_kC^{(3)})^2(i_kB) + 840\partial\Lambda^{(1)}(i_kC^{(3)})^3. \end{aligned} \tag{3.34}$$

On the other hand, the massless part of the transformation of $C'^{(9)}$ is obtained from (2.19) via the dimensional reduction. Using (3.33), and if we identify $\hat{\Omega}^{(9)}$ with $\Omega^{(8)}$ as

$$(i_k\Omega^{(8)}) = -(i_{\hat{k}}\hat{\Omega}^{(9)})_{\mu_1\cdots\mu_7 8}, \tag{3.35}$$

the gauge transformation of $C'^{(9)}$ completely agrees with the r.h.s of (3.34) ! Thus, we conclude that the WZ action of the KK-8A brane (which we regard as a kind of D-8-brane) is certainly obtained from that of the M-9-brane on this dimensional reduction. Therefore, on the double dimensional reduction, the relations of branes in Figure 1 is consistent from WVEA point of view.

3. 3 The dimensional reduction along the transverse direction

Finally, we discuss the dimensional reduction of S^{WZ}_{M9} along the single direction transverse to the M-9-brane, and show that the NS-9A brane WZ action S^{WZ}_{NS9A} is derived on this reduction.

In this case, we split $\hat{x}^{\hat{\mu}}$ into (x^μ, y) $(\mu = 0, 1, \cdots, 8, z)$, and dimensionally reduce the 11D fields along y. Naively, the SUGRA describing the bulk might be expected to be the same as the KK-8A brane case, since so are the relations of the target-space fields in 11D with those in 10D except for the replacement z or $8 \to y$. However, a certain

[¶]Note that the notation in refs.[30][25] is changed as $\Lambda^{(2)} \to -\Lambda^{(2)}$, $\Lambda \to -\Lambda^{(1)}$, $\Sigma^{(6)} \to \Omega^{(6)}$ and $\tilde{\Lambda} \to \Lambda^{(5)}$ (in addition to $C^{(3)} \to -C^{(3)}$, $B \to -B$).

[‖] $\delta C^{(1)}$ corresponds to the coordinate transformation in the direction parametrized by $\hat{x}^8$. We do not discuss the transformation with respect to $\Lambda^{(0)}$ here.

truncation procedure is needed in this case because the obtained brane is a 9-brane in 10 dimensions, i.e. a spacetime-filling brane. The famous example is the case of D-9-branes in IIB theory[3]; the full field content of IIB theory is $\{g_{\mu\nu}, \phi, B, B^{(10)}, C^{(0)}, C^{(2)}, C^{(4)}, C^{(10)}\}$, but due to the conservation law of the charge associated with $C^{(10)}$, the number of D-9-branes is constrained to be 32, and that orientifolding by the worldsheet parity is needed. In this orientifold construction, the IIB multiplet is truncated to that of the N=1 SUGRA with the condition $\{B = C^{(0)} = C^{(4)} = B^{(10)} = 0\}$, and Type I string theory arises. Then, starting from this case and using S- and T-duality, the authors of ref.[10] have argued that if the NS-9A branes exist, their number is also constrained to be 32, and that the following truncation condition is imposed on the RR fields of the IIA SUGRA:

$$C^{(1)} = C^{(3)} = 0 \tag{3.36}$$

$$C^{(9)} = 0. \tag{3.37}$$

In particular, the last condition is derived via T-duality from the condition that the IIB 10-form $C^{(10)}$ should vanish. However, this 10D theory still has a Killing isometry with the vector $k^\mu \equiv \hat{k}^\mu$. (Note that $\hat{k}^y = 0$.) Thus, precisely speaking, the 9-form potential which should arise as a T-dual of the IIB 10-form $C^{(10)}$ in this case is not the usual 9-form $C^{(9)}$ but the 9-form $C'^{(9)}$. Therefore, instead of (3.37), we set the truncation condition:

$$C'^{(9)} = 0. \tag{3.38}$$

Moreover, this condition, together with the duality relation (3.9) implies a further condition $m = 0$.

On the other hand, the truncation conditions on $i_k N^{(8)}$ and $i_k N^{(7)}$ have not been discussed before, but in this case we can infer it from the latter of the two duality relations (2.15). Dimensionally reducing the relation along y and substituting the truncation conditions (3.36) and (3.37) for it, we have the condition $\partial(i_k N^{(8)}) = 0$, which essentially means $i_k N^{(8)} = 0$. We note that no truncation condition is imposed on $i_k N^{(7)}$.

As for the worldvolume fields, a vector field and an 8-form $\omega^{(8)}_{i_1\cdots i_8}$ appears, coming from $\hat{b}_i$ and the 8-form $\hat{\omega}^{(8)}_{i_1\cdots i_8}$, respectively. In addition, a scalar field $Y(\xi)$ arises from the embedding of ξ^i in the transverse coordinate $\hat{x}^y = y$. We identify $\hat{b}_i = d^{(1)}_i$ for $i = 0, \cdots, 8$ and $Y(\xi) = c^{(0)}$ where $d^{(1)}_i$ is a vector field and $c^{(0)}$ is a scalar field, both defined in ref.[10]. We note that after the truncation, the dimensional reduction of $\hat{A}_i$ is deduced from (3.11) as $\hat{A}_i = A_i \equiv -|k|^{-2}\partial_i X^\mu k_\mu$.

Then, the WZ action of the 9-brane S'''^{WZ} obtained from that of the M-9-brane (2.24) is written as

$$S'''^{WZ} = \int d^9\xi \epsilon^{i_1\cdots i_9}[-\frac{1}{9!}(i_k B^{(10)}) + \frac{1}{6!}(i_k N^{(7)})\partial c^{(0)}\partial d^{(1)}$$

$$-\frac{1}{2\cdot 5!}(i_kB^{(6)})\{(\partial d^{(1)})^2+\partial d^{(1)}(i_kB)\partial c^{(0)}\}+\frac{1}{2\cdot 3!}B(\partial d^{(1)})^3\partial c^{(0)}$$
$$+\frac{1}{3!}A(i_kB)(\partial d^{(1)})^3\partial c^{(0)}+\frac{1}{4!}A_i(\partial d^{(1)})^4+\frac{1}{8!}\partial\omega^{(8)}]_{i_1\cdots i_9}. \tag{3.39}$$

This is exactly the same as the WZ action of the NS-9A brane given in ref.[10] !** Thus, we conclude that on the direct dimensional reduction of the M-9-brane, the relations in Fig.1 is also consistent from WVEA point of view.

The explicit correspondence of the terms in the actions is as follows: Each of the first three terms of the M-9-brane WZ action (2.24) only gives each of the first three terms of the NS-9A brane WZ action (3.39), respectively. The fourth term (including $\hat{C}$) in (2.24) gives the fourth term (including B) in (3.39), and the fifth term (including $\hat{A}_i$) in (2.24) gives the fifth and the sixth terms (including A_i) in (3.39). The last term in (2.24) gives the last term in (3.39), of course. All the other terms vanish, mainly due to the truncation conditions. We note that it holds $(i_kB)^2=(\partial c^{(0)})^2=0$ in the action.

4 Summary and discussion

In this paper we have shown that on dimensional reductions along three different directions, the Wess-Zumino action of the M-9-brane respectively gives those of the D-8-brane, the KK-8A brane (which we regard as a D-8-brane in some special background) and the NS-9A 9-brane, the last two of which were obtained via dualities. Therefore, we conclude that the relation of p-branes for $p\geq 8$, or of the M-9-brane with the other branes, proposed previously[4][23][10][24] (and represented in Fig.1) is consistent from the viewpoint of worldvolme actions.

Now, we discuss the massive part of the KK-8A brane WZ action. In section 3.2 we have set $m=0$ and discussed the massless part of the actions, since only massless part of S^{WZ}_{KK8A} has been discussed in ref.[25]. However, the mass parameter m corresponds nearly to the field strength dual of the 9-form $C^{'(9)}$ (and hence $N^{(9)}$) (see (3.10)), so setting the mass parameter to be zero essentially implies some trivial configuration of the background 9-form potential $C^{'(9)}$ (or $N^{(9)}$). Thus, the massive part should also be taken into account if one want the action of the KK-8A brane in a more general background. So, as another result of this paper, we propose the massive part of S^{WZ}_{KK8A} which was not obtained previously; It is derived from the dimensional reduction of $\frac{m}{5!}\hat{b}_{i_1}(\partial\hat{b})^4$, the massive part of S^{WZ}_{M9}, as

$$S^{WZ}_{KK8A}|_{\text{massive}}=\int d^9\xi\epsilon^{(8)i_1\cdots i_8}\frac{m}{5!}\{\omega^{(0)}(\partial\omega^{(1)})^4-4(\partial\omega^{(0)})\omega^{(1)}(\partial\omega^{(1)})^3\}_{i_1\cdots i_8}. \tag{4.1}$$

**To make the complete correspondence with the action of ref.[10], we have only change our notation as $C^{(3)}\to -C^{(3)},\ B\to -B$ and $A_i\to -A_i$

On the other hand, the action of the KK-8A brane (3.15) is derived from that of the D-7-brane via S- and T-dualities defined in ref.[25], but the duality relations of target-space fields discussed in ref.[25] are only the massless parts of them. They should be generalized to those in some massive background (like the usual case done in ref.[12][26]), so that the massive part of the KK8A brane WZ action (4.1) is reproduced, but we do not discuss them further here.

Here we would like to note the following two things: One is that trivial configuration of $C'^{(9)}$ does not mean the inconsistency of the action S^{WZ}_{KK8A} but merely implies some loss of generality of the background. (A similar case happens if one consider the worldvolume action of a D-8-brane in a *massless* background.) Another is that the consistency check of the field redefinition relation (3.24) we have made in section 3.2 is not a trivial one. The reason is that even if the configuration of $C'^{(9)}$ is trivial, the gauge transformations of $C'^{(9)}$ and the other fields do not become trivial; They are not affected except for the setting $m = 0$.

Finally, we would like to give a comment on the relation of the existence of the KK-8A brane with spacetime superalgebras. In ref.[24][25], it is argued that the KK-8A brane is a brane not predicted by the IIA spacetime superalgebra. However, we do not agree with this argument. Our objection is based on the fact that the existence of the M-9-brane, the 11D origin of the KK-8A brane, is suggested by the 11D superalgebra: Suppose we denote p-form central charges of superalgebras as $Z^{(p)}_{\mu_1\cdots\mu_p}$. Then, in the massive 11D theory where an isometry direction parametrized by z is assumed, the existence of an M-9-brane extending to the directions of $\hat{x}^1, \cdots \hat{x}^8$ and z corresponds to a non-vanishing central charge $\hat{Z}^{(2)}_{09} = \hat{\widetilde{Z}}^{(9)}_{01\cdots 8z}$ of the 11D superalgebra[4], where $\widetilde{Z}^{(D-p)}_{\mu_1\cdots\mu_{D-p}}$ indicates the dual central charge of $Z^{(p)}$ in a D-dimensional superalgebra. If the M-9-brane is dimensionally reduced along z, it gives a D-8-brane and the D-8-brane has a corresponding charge $\widetilde{Z}_{01\cdots 8} \equiv \hat{\widetilde{Z}}^{(9)}_{01\cdots 8z}$ of the D=10 superalgebra of the usual massive IIA theory. In the same way, if the M-9-brane is dimensionally reduced along x^8, it gives a KK-8A brane (as a kind of D-8-brane), and the KK-8A brane should have a corresponding charge $\widetilde{Z}'_{01\cdots 7z} \equiv -\hat{\widetilde{Z}}^{(9)}_{01\cdots 8z}$ of the D=10 superalgebra of *another* massive IIA theory with an isometry direction. (The prime ' of $\widetilde{Z}$ implies "another".) Therefore, we argue that, at least only in the case of KK-8A brane the information of the existence of the brane is included in the D=10 spacetime superalgebra.

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